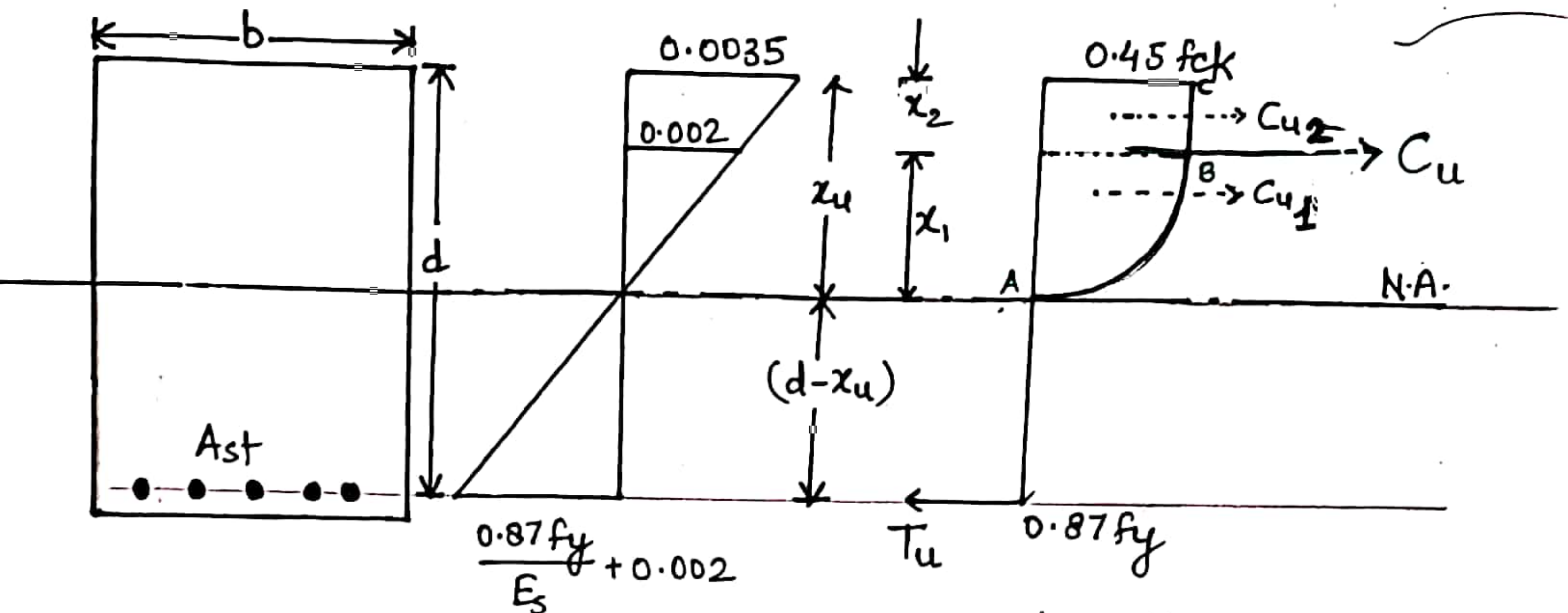
Analysis of a Singly Reinforced Beam —



(a) Strain Diagram (b) Stress Diagram

Strain Distribution —

i) Strain at N.A. = 0

ii) Max. strain in concrete = 0.0035

iii) Strain at constant stress of 0.67 fck = 0.002

iv) Max. strain in steel = $\frac{0.87 f_y}{E_s} + 0.002$

Stress Distribution —

Point A to B stress distribution parabolic होता है उसके बाद B से C Linear हो जाता है।

i) Stress at NA (point A) = 0

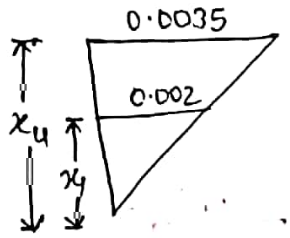
ii) Stress at 0.002 strain (point B) = $\frac{0.67 \, fck}{1.5} = 0.45 fck$

iii) Stress at 0.0035 strain (extreme fibre) (point C) = 0.45 fck

iv) stress in steel bars = 0.87 fy

Analysis :

In strain Diagram,



समरूप त्रिभुज (similar Triangle) के नियम से,

$$\frac{0.0035}{x_u} = \frac{0.002}{x_1}$$

$$x_1 = \frac{0.002\ x_u}{0.0035}$$

$$\boxed{x_1 = 0.57\ x_u} \quad or, \quad \boxed{x_1 = \frac{4}{7} x_u}$$

$$\because \quad x_1 + x_2 = x_u$$

$$x_2 = x_u - x_1$$

$$x_2 = x_u - 0.57\ x_u$$

$$\boxed{x_2 = 0.43\ x_u} \quad or, \quad \boxed{x_2 = \frac{3}{7} x_u}$$

So, Depth of Parabolic Part (x_1) = $0.57\ x_u$

Depth of Rectangular Part (x_2) = $0.43\ x_u$

Area of stress block = Area of Parabolic Part (A_1) + Area of Rectangular Part (A_2)

$$\boxed{A = A_1 + A_2}$$

$$A_1 = \frac{2}{3} \times x_1 \times 0.45\ fck$$

$$A_1 = \frac{2}{3} \times 0.57\, x_u \times 0.45\, f_{ck}$$

$$\boxed{A_1 = 0.171\, f_{ck} \cdot x_u}$$

$$A_2 = x_2 \times 0.45\, f_{ck}$$

$$A_2 = 0.43\, x_u \times 0.45\, f_{ck}$$

$$\boxed{A_2 = 0.194\, f_{ck} \cdot x_u}$$

$$A = A_1 + A_2$$

$$A = (0.171 + 0.194)\, f_{ck} \cdot x_u$$

$$\boxed{A = 0.36\, f_{ck} \cdot x_u}$$

Compressive force (C_u) = $C_{u_1} + C_{u_2}$

C_{u_1} = Comp. force in parabolic part

C_{u_2} = Comp. force in rectangular part

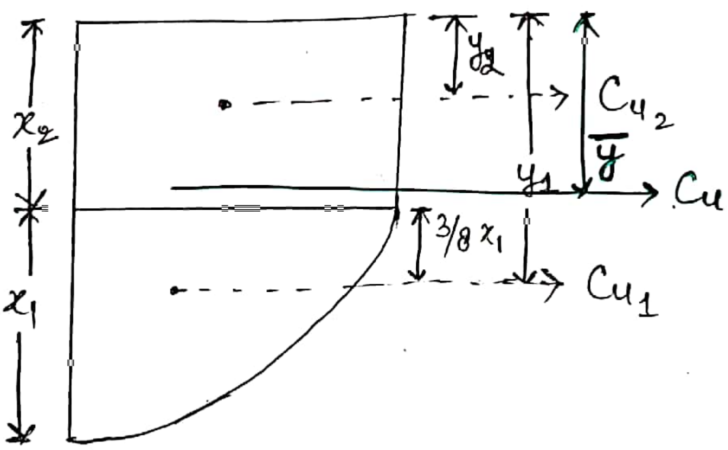
Comp. force = area of stress block x width of beam

$$\boxed{C_u = 0.36\, f_{ck} \cdot x_u \cdot b}$$

or,

$$C_u = 0.171\, f_{ck} \cdot x_u \cdot b + 0.194\, f_{ck} \cdot x_u \cdot b$$

$$\boxed{C_u = 0.36\, f_{ck} \cdot x_u \cdot b}$$



$$\boxed{y_2 = \frac{x_2}{2}}$$

$$\boxed{y_1 = x_2 + \frac{3}{8} x_1}$$

According to Varignon's Theorem,

$$C_{u_1} \times y_1 + C_{u_2} \times y_2 = C_u \times \bar{y}$$

$$\bar{y} = \frac{C_{u_1} \times y_1 + C_{u_2} \times y_2}{C_{u_1} + C_{u_2}}$$

$$\bar{y} = \frac{C_{u_1} \times y_1 + C_{u_2} \times y_2}{C_u}$$

By putting values of C_u, C_{u_1}, C_{u_2}, y_1 and y_2 and solving it.

we get, $\boxed{\bar{y} = 0.42\, x_u}$

Tensile force (T_u) = stress × Area

$$\boxed{T_u = 0.87\, f_y \cdot A_{st}}$$

Lever Arm, $Z = (d - \bar{y})$

Lever Arm = Vertical distance b/w C_u & T_u

$$\boxed{Z = d - 0.42\, x_u}$$