A. Maths. I (B)

Unit. 1

Chapter.

Definite Integrals.

(निश्चित समाकलन)

जब किसी फलन का समाकलन, किन्हीं दो निश्चित सीमाओं के बीच ज्ञात किया जाता है तो उसे निश्चित समाकलन कहते है।

$$\int_a^b f(x)\cdot dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

1. $\int_0^1 \frac{dx}{x^2+2x+10}$

$$= \int_0^1 \frac{dx}{x^2+2x+1+9}$$

$$= \int_0^1 \frac{dx}{(x+1)^2+3^2}$$

$$= \left[\frac{1}{3}\tan^{-1}\frac{x+1}{3}\right]_0^1$$

$$= \frac{1}{3}\left(\tan^{-1}\frac{2}{3} - \tan^{-1}\frac{1}{3}\right)$$

2. $\int_0^\pi \sin^4 x\cdot dx$

$$= \int_0^\pi (\sin^2 x)^2\cdot dx$$

$$= \int_0^\pi \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \int_0^\pi \frac{1+\cos^2 2x - 2\cos 2x}{4}\cdot dx$$

$$= \frac{1}{4}\int_0^\pi \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right) dx$$

$$= \frac{1}{8}\int_0^\pi (3 - 4\cdot\cos 2x + \cos 4x)\, dx$$

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$$= \frac{1}{8}\left[3x - 4\cdot\frac{\sin 2x}{2} + \frac{\sin 4x}{4}\right]_0^{\pi}$$

$$= \frac{1}{8}\left[(3\pi - 0) - 2(\sin 2\pi - \sin 0) + \frac{(\sin 4\pi - \sin 0)}{4}\right]$$

$$= \frac{1}{8}\left[3\pi - 2(0-0) + \frac{(0-0)}{4}\right]$$

$$= \frac{1}{8}[3\pi] = \frac{3\pi}{8}$$

(3) $\int_0^{\pi} \sin^5 x \cdot dx$

$$= \int_0^{\pi} \sin^4 x \cdot \sin x \cdot dx$$

$$= \int_0^{\pi} (\sin^2 x)^2 \cdot \sin x \cdot dx$$

$$= \int_0^{\pi} (1-\cos^2 x)^2 \cdot \sin x \cdot dx$$

$$\left.\begin{aligned} \cos x &= t \\ -\sin x \cdot dx &= dt \end{aligned}\right\} \begin{matrix} 1 \\ -1 \end{matrix}$$

$$= \int_1^{-1} (1-t^2)^2 \cdot (-dt)$$

$$= \int_{-1}^{1} (1+t^4-2t^2)\, dt$$

$$= 2\int_0^1 (1+t^4-2t^2)\cdot dt$$

$$= 2\left[t + \frac{t^5}{5} - 2\cdot\frac{t^3}{3}\right]_0^1$$

$$= 2\left[1+\frac{1}{5}-\frac{2}{3}\right] = 2\left(\frac{8}{15}\right)$$

$$= \frac{16}{15}$$

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(4) $\int_0^{\pi/2} x^2 \cdot \sin^2 x \cdot dx$

$$= \int_0^{\pi/2} x^2 \cdot \left(\frac{1-\cos 2x}{2}\right) dx$$

$$= \frac{1}{2}\int_0^{\pi/2} x^2 \cdot dx - \frac{1}{2}\int_0^{\pi/2} \underset{\mathrm{I}}{x^2} \cdot \underset{\mathrm{II}}{\cos 2x} \cdot dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3}\right]_0^{\pi/2} - \frac{1}{2}\left[x^2 \cdot \frac{\sin 2x}{2} - \int 2x \cdot \frac{\sin 2x}{2} \cdot dx\right]_0^{\pi/2}$$

$$= \frac{\pi^3}{48} - \frac{1}{4}\left[x^2 \cdot \sin 2x - 2\left\{x \cdot \left(\frac{-\cos 2x}{2}\right) - \int 1 \cdot \left(\frac{-\cos 2x}{2}\right) \cdot dx\right\}\right]_0^{\pi/2}$$

$$= \frac{\pi^3}{48} - \frac{1}{4}\left[x^2 \cdot \sin 2x + x \cdot \cos 2x - \frac{\sin 2x}{2}\right]_0^{\pi/2}$$

$$= \frac{\pi^3}{48} - \frac{1}{4}\left[(0-0) + \left(-\frac{\pi}{2} - 0\right) - \frac{(0-0)}{2}\right]$$

$$= \frac{\pi^3}{48} - \frac{1}{4}\left(-\frac{\pi}{2}\right)$$

$$= \frac{\pi^3}{48} + \frac{\pi}{8}$$

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