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A. Maths. I (B)

Unit 1

Chapter.

Definite Integrals.

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Properties of Definite Integrals.

1. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

2. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, $(a < c < b)$

4. $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

5. $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

6. $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$, $f(x)$ is even function

7. $\int_{-a}^a f(x)\,dx = 0$, $f(x)$ is odd function.

8. $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$, $f(2a-x) = f(x)$

9. $\int_0^{na} f(x)\,dx = n\int_0^a f(x)\,dx$, $f(na-x) = f(x)$

(1) $\int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}}\,dx = \frac{\pi}{4}$

Solution. $\int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}}\,dx$

$$= \int_0^{\pi/2} \frac{1}{1+\sqrt{\frac{\sin x}{\cos x}}}\,dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad \text{--- (1)}$$

$$= \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2 - x)}}{\sqrt{\cos(\pi/2 - x)}+\sqrt{\sin(\pi/2 - x)}}\,dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad \text{--- (2)}$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx + \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$= \int_0^{\pi/2} 1 \cdot dx$$

$$= [x]_0^{\pi/2}$$

$$= \pi/2$$

$$I = \pi/4$$

(2) $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}} = \pi/4$

$$\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$$

$$\left.\begin{aligned} x &= a\sin\theta \\ dx &= a\cos\theta \cdot d\theta \end{aligned}\right] \begin{aligned} &x=0,\ \theta=0 \\ &x=a,\ \theta=\pi/2 \end{aligned}$$

$$= \int_0^{\pi/2} \frac{a\cos\theta \cdot d\theta}{a\sin\theta+\sqrt{a^2-a^2\sin^2\theta}}$$

$$= \int_0^{\pi/2} \frac{a\cos\theta \cdot d\theta}{a\sin\theta + a\cos\theta}$$

$$I = \int_0^{\pi/2} \frac{\cos\theta \cdot d\theta}{\sin\theta+\cos\theta} \quad \text{(1)}$$

$$= \int_0^{\pi/2} \frac{\cos(\pi/2-\theta)}{\sin(\pi/2-\theta)+\cos(\pi/2-\theta)}\, d\theta$$

$$= \int_0^{\pi/2} \frac{\sin\theta \cdot d\theta}{\cos\theta+\sin\theta} \quad \text{(2)}$$

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta+\cos\theta}\, d\theta + \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta+\sin\theta}\, d\theta$$

$$= \int_0^{\pi/2} \frac{\cos\theta+\sin\theta}{\sin\theta+\cos\theta}\, d\theta$$

$$= \int_0^{\pi/2} 1 \cdot d\theta$$

$$= [\theta]_0^{\pi/2}$$

$$= \pi/2$$

$$I = \pi/4$$

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(3) $\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2} \log 2$

Solution. $I = \int_0^{\pi/2} \log \sin x . dx$ —— (1)

$= \int_0^{\pi/2} \log \sin (\frac{\pi}{2} - x) \, dx$

$= \int_0^{\pi/2} \log \cos x . dx$ —— (2)

$\Rightarrow 2I = \int_0^{\pi/2} \log \sin x . dx + \int_0^{\pi/2} \log \cos x . dx$

$= \int_0^{\pi/2} (\log \sin x + \log \cos x) \, dx$

$= \int_0^{\pi/2} \log (\sin x . \cos x) \, dx$

$= \int_0^{\pi/2} \log \left(\frac{\sin 2x}{2} \right) dx$

$= \int_0^{\pi/2} \log \sin 2x . dx - \int_0^{\pi/2} \log 2 . dx$

$\left. \begin{array}{l} 2x = t \\ 2dx = dt \end{array} \right] 0, \pi$

$= \frac{1}{2} \int_0^{\pi} \log \sin t . dt - \log 2 \int_0^{\pi/2} 1 . dx$

$= \frac{1}{2} . 2 \int_0^{\pi/2} \log \sin t . dt - \log 2 [x]_0^{\pi/2}$

$= \int_0^{\pi/2} \log \sin x . dx - \frac{\pi}{2} \log 2$

$\Rightarrow I = -\frac{\pi}{2} \log 2$